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Optimal design for hybrid rocket engine for air launch vehicle

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Abstract

A feasibility study and the optimal design was conducted for the application of a hybrid motor with HTPB/LOX combination to the first stage of an air launch system. The feasibility analysis showed that the hybrid motor could successfully be used as a substitute for the solid rocket motor of the first stage of the Pegasus XL if the average specific impulse (I_{sp}) of the hybrid motor could be approximately 350sec. And the optimal design of the hybrid motor was carried out for a given mission requirement with selected design variables: number of ports, initial oxidizer flux, combustion chamber pressure, and nozzle expansion ratio. The design results show the hybrid motor can be successfully applicable to the air launching vehicle. And, the optimal design can meet physical constraints of length and diameter imposed by the mother plane installation for a 3.5kg nano-sat to the orbit located on the 200x1500km elliptic orbit plane. Also, it was found that the design results would provide nearly the same design configurations regardless of the selection of objective functions: either mass or length of the engine.

Keywords: Hybrid rocket; Air launch vehicle; Optimal design

1. Introduction

The launching demands for small satellites are rapidly growing with the advent of advanced communication technology and MEMS (micro-electro mechanical systems) technology. A source revealed that the total number of planned satellite launches would be more than about 700 by the year of 2008 and the half of the number would be occupied by small satellites [1]. The excessive demand for satellite launches will stimulate the launcher service provider to find an economic way of launching both large and small satellites simultaneously with a large booster. Thus, it is not surprising to seek alternative ways to launch a small satellite in a convenient time with less expensive cost by using the air launch vehicle. Also, the launch of Spaceship-1 developed by Rutan and coworkers could bring more public attention to the air launch vehicle(ALV) for one of the possible ways for private space tour [2].

Although many studies have been conducted for the air launch vehicle (ALV) since the late of 1950s, air-launching technology has many basic advantages but it is still a challenging one. This launch method can take advantage of the initial speed of the mother plane. And another advantage is in the choice of launching location. Since the mother plane can fly to any spot on the globe, there is great flexibility in choosing the launching spot. Also, the propellant mass of air launching vehicle could be dramatically reduced because this technology could take an advantage of high altitude launching. However, ALV has some disadvantages as well. For example, the physical configuration such as diameter and length of engine is severely limited by the size of the mother plane [3].

Although many attempts have been tried to develop an air launching vehicle by various countries, Pegasus XL by OSC (Orbital Science Corp.) is the only commercially available launching vehicle. This vehicle has a configuration of three-stage, solid propellant motor,

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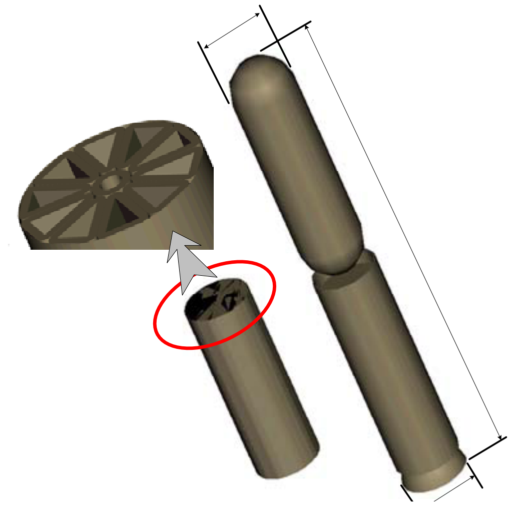


Fig. 1. Configuration design of hybrid rocket.

inertia guided and all composite winged boosters. Pegasus weighs 22ton with up to 450kg of payload and its launching altitude and launching speed were designed about 12km and Mach number of 0.8, respectively. Also, Pegasus has proved its commercial availability by showing about 80% of successful launching rate. A specially modified Lockheed L-1011 carrier has carried aloft the Pegasus XL [4].

2. Hybrid engine design code and validation

Even though the hybrid rocket engine has gained the spotlight recently, no public design data is available to the authors' knowledge. Thus, an in-house design code was developed to make a preliminary design for a hybrid rocket engine. Fig. 1 depicts the rocket configuration and design procedure implemented in the design code. Details of the design process and input variables are described in reference 5. A wagon wheel grain is used in the design code because this configuration is known to show excellent features in charging efficiency and fuel sliver fraction [6]. It should be noted that the fuel sliver can be easily torn off at the end of burning time and pass through the combustion chamber with the combustion gas if the structural strength of solid fuel is not strong enough to withstand thermal stresses. When this happens, fuel may block the nozzle throat and deteriorate rocket performance or even lead to explosion. However, this study assumed that the fuel structure is strong enough and can be completely consumed without making any fuel debris in the combustion chamber.

Fig. 2 shows the flow chart of sizing the hybrid engine with input variables from mission requirements. Velocity increment (ΔV), payload mass (M_{pay}), and fuel burning time (t_{b}) are given as the mission requirements to be achieved in engine design. Six variables are chosen as design variables: number of ports (N), combustion chamber pressure (P_{c}), initial oxidizer flux (G_{oxi}), nozzle expansion ratio (e), average Oxidizer/Fuel ratio (OF_{avr}), and nozzle divergence angle(θ_{n}).

The combination of oxidizer and fuel is the HTPB/LOX system because many studies have proved it can provide higher specific impulse (I_{sp}) up to 330sec without the addition of any energetic materials to propellants [7]. It is also well known that the combination of several technologies could improve the rocket performance parameters further and the average specific impulse can be an order of 350sec or so [7].

The thermodynamic properties of combustion gas, such as specific heat ratio, flame temperature, and average molecular mass, were calculated by using NASA–Lewis equilibrium code [8]. As for the regression rate of the selected propellant combination, the following expression was used [6]:

$$\dot{r} = 2.0 \times 10^{-5} G_{\mathrm{ox}}^{0.75} L_{\mathrm{p}}^{-0.15} \tag{1}$$

It is not surprising that oxidizer mass flux was assumed constant and the consequent O/F ratio varies during the combustion. Since the pressurized feeding system is relatively suitable for a small rocket system such as the air launch vehicle, the oxidizer was assumed to be fed into fuel grain by pressurized system.

The design can be initiated by guessing the ratio of structure mass to empty mass (κ), and the average specific impulse (Isp_{avr}) in the code. Mass distribution for propellant and inert structure can be evaluated by using Eq. (2) based on the guessed values and mission requirements if the specific impulse and O/F ratio are fixed as in the solid and liquid rocket system.

$$M_{\mathrm{ox}} = OF_{\mathrm{avr}} M_{\mathrm{fuel}} \tag{2a}$$

$$M_{\mathrm{o}} / M_{\mathrm{f}} = \exp\left(\frac{\Delta V}{Isp_{\mathrm{avr}} \cdot g} \right) \tag{2b}$$

However, O/F ratio and specific impulse are not fixed in a hybrid rocket because O/F ratio varies continuously due to the increase in fuel mass rate ($\dot{M}_{\mathrm{fuel}}$) during the combustion even if the oxidizer mass rate is

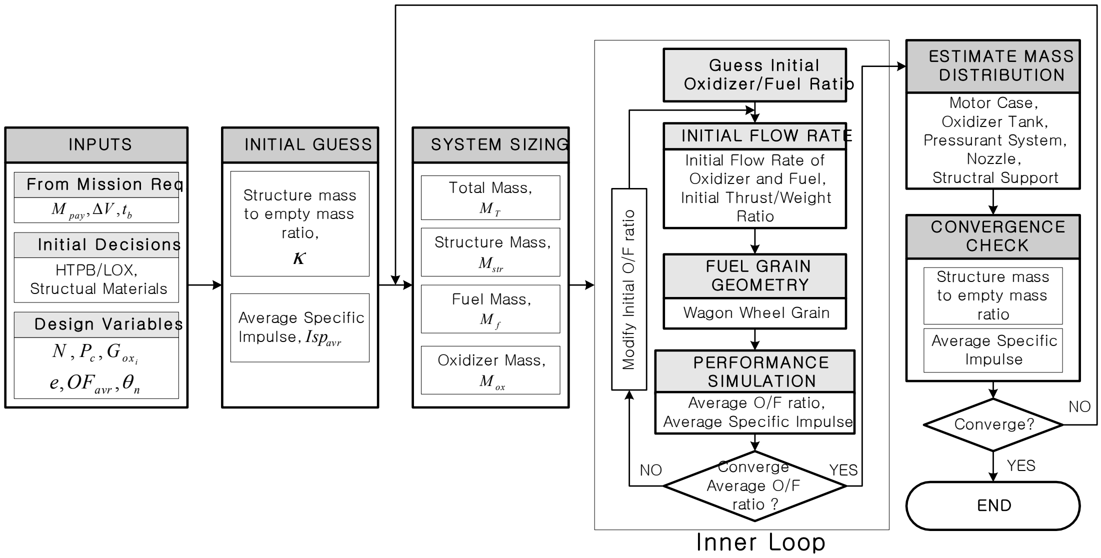


Fig. 2. Flow chart for hybrid rocket design process [5].

fixed. So, the evaluation of mass distribution in hybrid rocket needs the definition of average value of specific impulse and O/F ratio for exact calculation for mass distribution design process. By defining O/F ratio as in Eq. (3),

$$\dot{M}_{\text{fuel}} = \dot{M}_{\text{ox}} / OF \tag{3}$$

Fuel mass can be determined by integrating Eq. (3) if $\dot{M}_{\text{ox}}$ remains constant:

$$M_{\text{fuel}} = M_{\text{ox}} \frac{1}{t_{\text{b}}} \int_0^{t_b} \frac{1}{OF} dt \tag{4}$$

And comparing the Eq. (2a) with Eq. (4) yields the new definition of an average OF ratio as

$$\frac{1}{OF_{\text{avr}}} = \frac{1}{t_{\text{b}}} \int_o^{t_b} \frac{1}{OF} dt \tag{5}$$

Then the mass ratio of initial empty mass to final mass can be written as

$$M_{\text{o}} / M_{\text{f}} = \exp\left(\int_0^{\Delta V} \frac{1}{Isp \cdot g} dV \right) \tag{6}$$

Also, the average specific impulse can be expressed by comparing Eq. (2b) with (6) as

$$\frac{1}{Isp_{\text{avr}}} = \frac{1}{\Delta V} \int_o^{\Delta V} \frac{1}{Isp} dV \tag{7}$$

It is very interesting to see that the average of specific impulse and average O/F ratio can be defined as the time average of the reciprocal of each parameter rather than the conventional average. Once the average is evaluated, mass distribution can be determined by using Eq. (2). Thus, the inner loop in the flow chart can determine initial oxidizer mass flow rate, fuel grain configuration and average performance with guessed O/F ratio until the calculated average O/F ratio converges to the specified value as a design variable. As mentioned previously, the average value can be obtained by using Eq. (5), (7) with a given burning time. The updated value of guessed initial O/F ratio is calculated by linear interpolation method in the iteration.

At the exit of the inner loop, the structural mass distribution is estimated in detail: engine case, oxidizer tank, pressure system, nozzle, and structural supports. Then, the ratio of structure mass to empty mass (κ) is calculated with the estimated structure mass. Finally, the average specific impulse (Isp_{avr}) and mass ratio κ are tested and updated by using a fixed point iteration method. This iteration constructing the outer loop can make the design process

Table 1. Lower and upper bounds of design variables.

Variables	Lower bound	Upper bound
N	8	15
G_{ox} (kg/s/m^2)	100	350
P_c (Mpa)	1.0	5.0
e	4	20
OF_{avr}	2.1	2.7
θ_n (deg)	15	25

complete along with inner loop iteration. And the selection of the range of design variables is also an important aspect of the optimal design. The lower and upper bound of each design variable are selected from physical considerations. Especially, the chamber pressure and nozzle expansion ratio should be correlated to avoid nozzle flow separation [6]. Table 1 shows the design range of each design variables selected in this study.

To estimate total mass of rocket motor, every mass of all components should be taken into account for in the design process. These include combustion chamber, oxidizer tank, nozzle, oxidizer injector, polar boss, skirt, and TVC unit [6, 9]. By considering all these components, an in-house code was developed for systematic calculation of the engine performance and configuration. A developed code should be checked for its validity by the comparison design results with previously known design result. In reference 6, a hybrid engine is used for second stage booster to raise the payload from an altitude of 400km to 5,000km following Hohmann transfer orbit. From the mission analysis, rocket total mass is 12,000kg, and payload is 4,914kg. The consequent velocity increment is 1,721m/sec in this case. However, the final target of velocity increment is chosen as 1,893m/sec, 10% higher than the nominal value for safety margin. As mentioned previously, wagon wheel grain is used with HTPB/LOX propellant combination in the validation check.

Fig. 3 summarizes the comparison of design result of thrust and specific impulse with that in reference 6 and shows a very good agreement with reference values. As seen in table 2, the engine length in the design calculations also coincides with that in reference 6 for the same engine diameter. Thus, this result can prove that the design code developed in this study is a very good capability in designing hybrid rocket configuration and predicting performance.

Table 2. Comparison of design results with those in reference 6.

	N	L_M	D_M	Isp_{avr}	OF_{avr}
Ref.6	8	3.493	0.4675	323.7	2.26
Code	8	3.497	0.4625	324.6	2.37

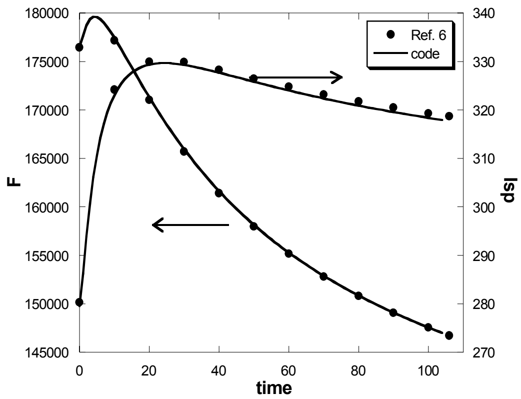


Fig. 3. Comparison of thrust simulation with that in reference 6.

3. Formulation of optimization and feasibility

Optimization techniques implement the design of a hybrid engine which satisfies all mission requirements and the design constraints imposed on the air launch vehicle. The optimal design problem describes the problem to find a set of design variables $\{N, G_{ox_i}, P_c, OF_{avr}, \theta_n, e\}$ which minimizes a design objective function (J), and the mathematical expression becomes

$$\min_{\{N, G_{ox_i}, P_c, OF_{avr}, \theta_n, e\}} J \tag{8}$$

with constraint function f_i

$$f_i\left(N, G_{ox_i}, P_c, OF_{avr}, \theta_n, e\right) \le 0, \quad i = 1, 2, ... \tag{9}$$

It is worth noting that the design process of rocket engine has been, in many cases, focused on seeking the design variables to minimize total mass or inert mass. And the minimization of rocket length or diameter is not a design issue in general. In air launching vehicle, however, the installation to the mother plane is also an important aspect to consider in the design process. Thus, both the rocket total mass and total length need to be considered as the design objective. Some geometric constraints such as rocket diameter (D_M) and nozzle exit diameter (D_N) are also imposed for an air launched vehicle because of the limited space of the mother plane. Thus, the constraints can be additionally defined as

$$f_1 = D_N - D_M \le 0 \tag{10}$$

$$f_2 = D_M - D_{upp} \le 0 \tag{11}$$

The constraints Eq. (10) - (11) represent the geometrical constraints that the rocket diameter must be larger than the nozzle exit diameter and is limited by the given upper bound simultaneously. Also, it is worth noting every design variables are mathematically real value but the number of ports is an integer. The operational range of chamber pressure in the design is selected based on the value in refs. 5, 6. Also, the variation of nozzle expansion ratio lies from 4 to 20.

The NPSOL package [11] is used for the optimization. NPSOL is a set of FORTRAN subroutines to calculate the optimal condition based on SQP (sequential quadratic programming) algorithm. And this package is suitable to minimize a smooth function subject to constraints. The optimal design problem denoted by Eq. (8) includes an integer variable N and it should be treated properly in the optimization process. In many design processes, integer variables are assumed as real numbers and the nearest integer is chosen [12] from the result as the optimal value. However, it does not always guarantee the optimal solution. An alternative way is to optimize the design process firstly with each fixed integer within the range of integer variable. Then, the optimal solution is chosen out of the locally optimized solutions for each fixed integer. This method can result in the true optimal value and hence is preferred if the range of integer variable is not too wide. Fortunately, since the specified range of the port number is small in this study, the current design of hybrid rocket utilizes the second method.

4. Feasibility of hybrid engine for air launch vehicle

Hybrid rocket has some different features from solid or liquid rocket in that physical configuration becomes bulky in volume and length. And, the performance such as thrust and specific impulse varies continuously during combustion as well. Thus, it is better to study the feasibility of the application of hybrid rocket to air launch vehicle as an alternative propulsion system of solid propellant rocket. To this end, the comparison is made between the configuration and performance of Pegasus XL and that of optimally designed hybrid rocket based on the same mission requirements. The following summarizes the mission analysis data and configuration of Pegasus XL from reference 4.

· Altitude change; 12km - 63km
· Velocity increment; 3172.4m/sec
· Payload mass of first stage; 5426kg
· Burning time; 64sec
· Average thrust; 620kN
· Total vehicle mass; 22,584kg
· Rocket diameter; 1.30 m
· Rocket length (first stage); 10.30m

The alternative hybrid rocket for the first stage of Pegasus XL is assumed to use the HTPB/LOX propellant combination along with wagon wheel configuration. The objective function is the minimization of rocket length in the optimal design process. Also, design constraints such as rocket diameter and nozzle exit diameter are taken into account in the optimization process. For the feasibility analysis of the hybrid rocket engine, the optimization was done to have a minimal rocket length with a constraint of rocket diameter.

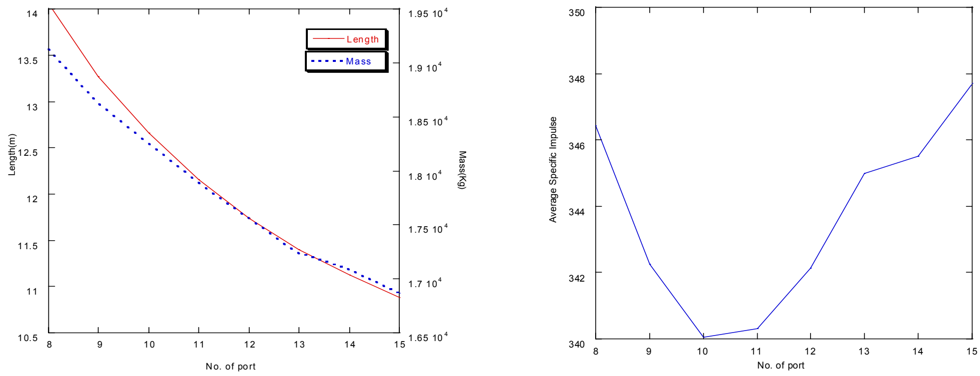
· Design Objective; $J = L_M$
· Constraints; $f_1 = D_N - D_M \le 0$
$f_2 = D_M - D_{upp} \le 0$

In the design process, the upper bound of rocket diameter (D_{upp}) was fixed as 1.6m, 20% larger than Pegasus since no converged solutions can be obtained in the case of the limit of 1.3 m. So, the upper limit of diameter is relaxed with 20% margin to evaluate the plausible design results. Table 3 summarizes the design result showing the variation of total mass and length over the number of ports. And the following are the design results for length minimization.

· Diameter; 1.6 m,
· Total length; 10.88 m,
· Total mass; 16865 kg,
· Average specific impulse 348 sec
· Average OF ratio; 2.49
· Initial oxidizer flux; 100.42 kg/sec/m2
· Chamber pressure; 1.4Mpa

Table 3. Design results for feasibility study of substituting the SRM of Pegasus XL.

No. Port	8	9	10	11	12	13	14
Diameter(m), D_M	1.6	1.6	1.6	1.6	1.6	1.6	1.6
Nozzle Dia.(m), D_N	1.6	1.52	1.47	1.46	1.5	1.55	1.54
Min. Length(m)	14.05	13.26	12.67	12.16	11.74	11.40	11.13
Total Mass(Kg)	19128.4	18623.4	18247.7	17892.9	17555.8	17238.0	17086.2
Isp_{avr} (sec)	346.44	342.24	340.04	340.3	342.13	344.99	345.51
OF_{avr}	2.7	2.7	2.7	2.7	2.7	2.7	2.65
G_{ox_i} (kg/sec/m/m)	101.13	100	100	100	100	100.04	101.47
P_c (Mpa)	1.4	1.4	1.4	1.4	1.4	1.4	1.4
e	7.0	6.43	6.02	6.09	6.46	7.09	7.0
θ_n (deg)	23.7	25.14	25.39	25.34	25.09	24.67	24.70



(a) (b)

Fig. 4. Convergence history of average specific impulse in the optimization for length with length and diameter constraints.

The results show that the chamber pressure does not vary and becomes 1.4Mpa for the entire range of N. As seen in Fig. 4, the length and mass of the hybrid engine decrease linearly as N increases. And the minimum length is located at the upper bound of N, which coincides with the location of minimum mass. It is interesting to see that the total mass of the hybrid rocket in the result is 16865kg, which is about 34% less than Pegasus. Thus, a hybrid rocket engine could be possibly used as an alternative system to a solid rocket in Pegasus XL, even though the hybrid rocket is longer in length of 10.88m and larger in diameter. However, it should be noted that design results imply that only a high performance hybrid rocket can substitute currently available SRM in Pegasus only if the average specific impulse of hybrid engine is high enough up to about 350 sec. The maximum value of specific impulse in HTPB-LOX combination is reported as about 330sec. And the addition of energetic materials such as AP and Al powder may increase the specific impulse by about 10% [6]. Thus, the feasibility study here was done to check the possibility of substituting SRM in Pegasus with a hybrid rocket even though the average specific impulse of 350 sec is somewhat higher than the theoretical value of HTPB/LOX. Fig. 4-b shows the trajectory of the average specific impulse during the calculation. The minimum I_{sp} is calculated as 340 sec when N=10. The length, however, at minimum I_{sp} is 12.66 m, much longer than the minimum length of Pegasus even though the total mass is still far less than Pegasus.

5. Optimal design of hybrid rocket for air launch vehicle

The purpose of the optimization is to design a hybrid rocket engine for the first stage of an air launch vehicle to launch nano-size satellites. Fig. 5 shows the mission profile of the air launch vehicle currently under study [13]. Mission analysis of the launch vehicle reveals a nano-sat with 3.5kg of mass as payload and its orbit is located on the 200x1500km elliptic plane. The F-4E phantom II fighter is the candidate for the mother plane and the launch vehicle can be attached to the center body of the mother plane. Dimensions and configuration of ALV must be basically determined by considering the physical appearance of the auxiliary fuel tank of the mother plane. Based on the configuration data of the F-4E II, the auxiliary tank has dimensions of 7.11m in length, 0.87m in diameter, and 600gal in volume. Thus, ALV has to be designed at least to have a total length of 7.0m and a diameter of 0.6m or less to avoid any collisions to ground during take-off and landing. The maximum allowable length of the hybrid rocket engine is 5m to accommodate the second, third stages and nose fairing. As seen in Fig. 5, the initial launch speed from the mother plane is assumed as Mach number of 1.3 at an altitude of 12km. The combustion is completed at an altitude of 43km after 41 sec of burning time.

The mission analysis then provides design requirements for the design of the hybrid rocket engine for the first stage of ALV. According to the mission analysis, payload mass is 152.1kg and the velocity increment of 3,676.2m/sec must be achieved by the first stage. It is not surprising to find that the velocity increment (2463.7 m/sec) in Fig. 5 differs from the nominal velocity increment of 3,676.2 m/sec because the viscous and aerodynamic drag can increase the required velocity increment for the first stage of the hybrid rocket. Also, the HTPB/LOX system with wagon wheel configuration is adopted in the hybrid rocket engine. Since the optimal design is seeking a

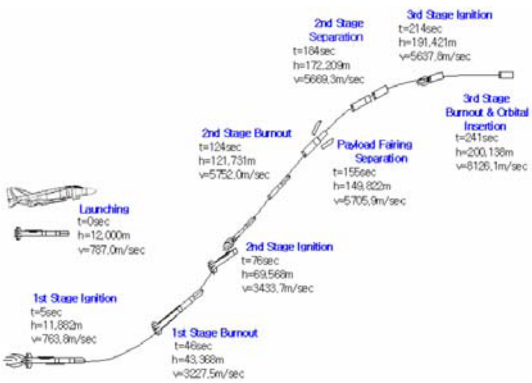


Fig. 5. Mission profile of pre-designed ALV.

configuration to install the hybrid rocket on the mother plane, the installation consideration, therefore, can be a primary factor in the design process. This consideration differs from other conventional concepts of launch vehicle design for minimizing the total vehicle mass [14]. In this study, the minimization of vehicle length and the minimization of vehicle total mass are considered as two different objective functions, respectively, and two design results are compared.

6. Length minimization with diameter constraint

The design has been done to minimize the vehicle length with the constraint that the vehicle diameter should be less than 0.6m. As mentioned previously in the mission analysis, the maximum allowable length of the first stage is limited to 5m. The range of design variables is specified as shown in table 1. The minimization can be formulated by the equations below:

· Design Objective; $J = L_{\mathrm{M}}$

· Constraints; $f_1 = D_{\mathrm{N}} - D_{\mathrm{M}} \le 0$

$f_2 = D_{\mathrm{M}} - D_{\mathrm{upp}} \le 0$

Here D_{upp} is the upper bound of the vehicle diameter specified as 0.6 m.

Table 4 depicts the design results over the range of N from 7 to 15. Fig. 6-a plots the design result of engine length and mass against number of ports N. As can be seen in figure, engine length is less than the design requirement of 5 m in the range of N over 11; the length converges to a certain value of about 4.8 m when N is 14 and 15. And, the minimum length is 4.8m at N of 15.

It is, however, interesting to find the behavior of engine mass showing a linear decrease in the range of N up to 12 and being approximately constant around 1058 kg. Thus, it can be summarized that the optimal design point may be in the range of N between 13 and 15 when two optimal values of engine design and mass are accounted simultaneously in the design process.

Fig. 6-b describes the behavior of vehicle mass along with the number of ports N. The design result shows the minimum mass of the first stage rocket to be 1057 kg at the number of port of 13. It is, however, also interesting to find that neighbor values of vehicle

Table 4. Minimization of engine length with a constraint of diameter.

No. Port	8	9	10	11	12	13	14
Diameter(m), D_M	0.6	0.6	0.6	0.6	0.6	0.6	0.6
Nozzle Dia.(m), D_N	0.46	0.45	0.45	0.45	0.47	0.47	0.48
Min. Length(m)	5.6	5.34	5.13	4.98	4.88	4.83	4.81
Total Mass(Kg)	1186.19	1144.4	1108.59	1077.82	1058.35	1057.06	1058.46
Isp_{avr} (sec)	283.3	284.06	285.98	288.7	291.20	291.36	291.68
OF_{avr}	2.53	2.41	2.26	2.12	2.1	2.1	2.1
G_{ox_i} (Kg/sec/m^2)	100	100	100	100	105.06	115.7	126.59
P_c (Mpa)	1.4	1.4	1.4	1.4	1.4	1.4	1.4
e	7.87	7.62	7.68	8.11	9.02	9.23	9.45
θ_n (deg)	19.95	20.08	19.88	19.27	18.31	18.09	17.87

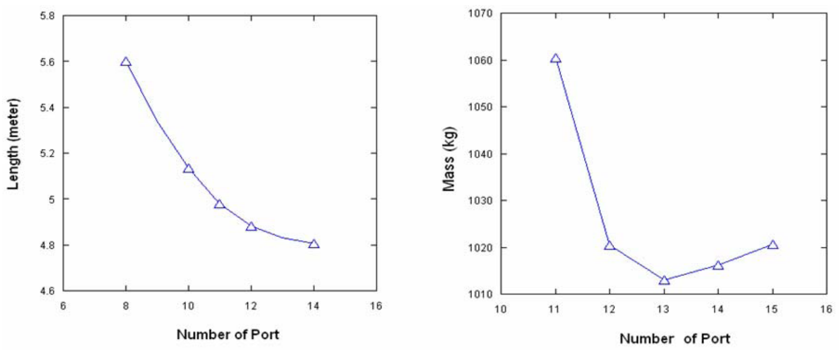


Fig. 6. Convergence history of mass and length in the optimization for mass.

mass at the number of ports of 12 and 14 have no discernible differences from the minimum mass showing 1058.3 and 1058.4 kg, respectively. Thus, the optimum design for vehicle length with diameter constraint reveals that the minimum vehicle mass may be near the location of N of 13. It is worth noting that the average specific impulse at the optimal length condition shows about 291 sec.

7. Mass minimization with length and diameter constraints

The result of optimal design for minimum engine length may provide the criterion if the hybrid rocket could be applied to the first stage of ALV without causing any installation problem to the mother plane. However, it would be also worth formulating a conventional type of optimal design problem considering the vehicle mass as the objective function coupled with geometrical constraints such as vehicle length and diameter. This optimization can be summarized with the formulation as:

· Design Objective; $J = L_M$

· Constraints; $f_1 = D_N - D_M \le 0$

$f_2 = D_M - D_{upp} \le 0$

$f_3 = L_M - L_{upp} \le 0$

This optimization should be performed over a certain range of N, where the geometrical constraints could be satisfied. Table 4 shows that the number of ports N of 11 is the lower bound to be considered in the mass optimization with geometrical constraints.

Table 5 summarizes the optimization results of the problem to minimize vehicle mass. The minimum vehicle mass is calculated as 1013 kg at the number of ports N of 13. This N coincides with results obtained in the previous calculation for length minimization. And the physical configuration of the hybrid engine shows the length and diameter are 5.0m and 0.6m, respectively. It is instructive to note the average specific impulse is 302 sec at the minimum mass condition, 10 sec higher than the result for length minimization.

Table 5. Minimization of engine mass with constraints of diameter and length.

No. Port	11	12	13	14	15
Diameter(m), D_M	0.6	0.59	0.6	0.6	0.6
Nozzle Dia.(m), D_N	0.46	0.53	0.6	0.6	0.6
Length (m)	5	5	4.99	4.96	4.94
Total Mass(Kg)	1060.5	1020.4	1013.0	1016.1	1020.6
Isp_{avr} (sec)	291.4	298.6	302.2	301.8	301.5
OF_{avr}	2.123	2.1	2.1	2.1	2.1
G_{ox_i} (Kg/sec/m^2)	100.0	100.0	100.2	110.3	119.9
P_c (Mpa)	1.4	1.4	1.4	1.4	1.4
e	8.8	12.0	15.1	15.6	15.6
θ_n (deg)	17.9	15	15	15	15

Table 6. Summary of design results for ALV; Length vs. mass minimization.

	Length	Mass
No. Port	12-14	13
Length (m)	4.8	5.0
Total Mass(Kg)	1058	1012
Diameter(m)	0.6	0.6
Isp_{avr} (sec)	291.0	301.7

8. Length optimization vs. mass optimization

Table 6 shows the comparison of optimization result for engine length with that for engine mass optimization. Basically, two design results do not show discernible differences in the overall configuration of the hybrid engine except mass and average specific impulse. This may indicate that any approaches can lead to the successful design with the optimum configuration. As seen in table 6, the result for mass minimization provides the minimum mass at the expense of higher specific impulse. Also, the optimization for minimum mass shows more relevant result for real rocket design than that for minimum length since the total cost of ALV seems to be directly proportional to total mass rather than length. Hence, it is suggested for the design of a hybrid rocket for ALV that the length minimization with a constraint of diameter be first performed to check if the installation problem could be satisfied; then the mass can be minimized with the geometrical constraints over the feasible number of ports N.

9. Conclusion

The optimal design of an air launch vehicle was conducted by using a hybrid engine with a propellant combination of HTPB/LOX having a wagon wheel configuration. Prior to the optimal design for the air launch vehicle, the feasibility of the application of a hybrid engine was analyzed and optimization results were compared with Pegasus XL data of a solid propellant motor. The comparison shows that a hybrid engine having a specific impulse of up to 350 sec can successfully substitute for the solid rocket motor without causing big differences in physical configuration. Moreover, the optimization results for a hybrid engine can provide more improved performance than the solid rocket motor with less total vehicle mass to meet the same mission requirements of a solid rocket motor.

Also, two different methods were conducted simultaneously to find which approach is more efficient and easier to have the optimal design of hybrid rocket used for the first stage of air launch vehicle. The first approach is done to find the minimum mass, and the other is to minimize the engine length. Results showed that two approaches could provide nearly the same design results regardless of the selection of objective functions: mass or length of the engine.

Nomenclature

ΔV : Velocity increment (m/sec)
D : Diameter (m)
e : Nozzle expansion ratio

F : Thrust (N)
FW : Thrust to weight ratio
G : Propellant mass flux (kg/m^2-sec)
Isp : Specific Impulse (sec)
L : Length (m)
M : Vehicle mass (kg)
M_0 : Initial vehicle mass
N : Number of ports of wagon wheel grain
OF : Oxidizer to fuel ratio
P_c : Combustion chamber pressure (MPa)
$\dot{r}$: Regression rate (mm/sec)
t_b : Burning time (sec)
θ_n : Nozzle divergence angle
κ : Ratio of structure mass to empty mass

Subscripts

avr : Average value
i : Mnitial value
fuel : Fuel
final : Final value
M : Motor
N : : Nozzle exit
ox : Oxidizer
p : Fuel grain port
pay : Payload

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